AS & GS

Form 6

Vol 1

Part 4 – Arithmetic Series

1. B
2. A
3. B
4. A
5. D

6. (a)

$$S(n)=1457$$
$$\frac{31}{2}(a+77)=1457$$
$$a=17$$

(b)

$$T(31)=77$$
$$17+30d=77$$
$$d=2$$
$$\text{General term}=15+2n$$

7. (a)

$$S(16)=108$$
$$\frac{16}{2}[2(-12)+15d]=108$$
$$-24+15d=13.5$$
$$d=2.5$$

(b) general term $=-14.5+2.5n$

8.

$$S(n)=539$$
$$\frac{n}{2}(19+58)=539$$
$$n=14$$
$$d=2.5$$

$\therefore$ 14 terms

9. $T(n)=4n+7$

$T(1)=4+7=11$

$T(2)=4(2)+7=15$

$a=11$, $d=15-11=4$

$S(m)<3998$

$\frac{m}{2}[2(11)+(m-1)(4)]<3998$

$2m^2+9m-3998<0$

$m<42.517$

$\therefore$ greatest value of $m=42$

10. $T(1)=55-3(1)=52$

$T(2)=55-3(2)=49$

$a=52$, $d=49-52=-3$

$T(9)+T(10)+T(11)+\ldots+T(41)$

$=S(41)-S(8)$

$=\frac{41}{2}[2(52)+40(-3)]-\frac{8}{2}[2(52)+7(-3)]$

$=-328-332$

$=-660$

11. $T(1)=14-6=8$

$T(4)=14-6(4)=-10$

$T(7)=14-6(7)=-28$

$d=T(4)-T(1)=-18$

$n=\frac{19-1}{3}+1=7$

$\therefore$ $T(1)+T(4)+T(7)+\ldots+T(19)$

$=\frac{7}{2}[2(8)+6(-18)]$

$=-322$

12. $S(2)=60$

$$\frac{2}{2}(2a+d)=60$$

$$2a+d=60......(1)$$

$$S(5)=105$$

$$\frac{5}{2}(2a+4d)=105$$

$$2a+4d=42......(2)$$

$\therefore$ we have $d=-6$

$\therefore$ difference $=-6$

13. $T(4)+T(5)+\ldots+T(8)=615$

$$S(8)-S(3)=615$$

$$\frac{8}{2}(2a+7d)-\frac{3}{2}(2a+2d)=615$$

$$8(2a+7d)-3(2a+2d)=1230$$

$$10a+5d=1230$$

$$a+5d=123......(1)$$

$$T(7)=a+6d=115......(2)$$

$\therefore$ we have $a=163$, $d=-8$

$\therefore$ general term $=171-8n$

14. (a) $S(1)=2(1)^2-4(1)=-2$

$\therefore$ first term $=-2$

(b) $T(12)=S(12)-S(11)$

$$=2(12)^2-4(12)-[2(11)^2-4(11)]$$

$$=42$$

$\therefore$ 12th term $=42$

15. (a) $S(1)=(1)(3-10)=-7$

$\therefore$ first term $=-7$

(b) $S(2)=(2)[3(2)-10]=-8$

$$S(2)=T(1)+T(2)$$

$$-8=-7+T(2)$$

$$T(2)=-1$$

$$d=T(2)-T(1)$$

$$=-1-(-7)$$

$$=6$$

$\therefore$ general term $=-13+6n$

Part 6 - Application

1. (a)

$$A(3) = 25$$
$$a + 2d = 25 ......(1)$$
$$A(8) = 55$$
$$a + 7d = 55 ......(2)$$

$\therefore$ we have $a = 13$, $d = 6$

$$S(n) = \frac{n}{2}[2(13) + (n-1)(6)]$$
$$= 3n^2 + 10n$$

(b) $A(n) = 7 + 6n$

$$B(n) = 10^{6n+7}$$
$$\log B(n) = 6n + 7$$
$$\log B(n) = A(n)$$

$$\log[B(1)B(2)\ldots B(n)] \le 8000$$
$$\log B(1) + \log B(2) + \ldots + \log B(n) \le 8000$$
$$A(1) + A(2) + \ldots + A(n) \le 8000$$
$$S(n) \le 8000$$
$$3n^2 + 10n \le 8000$$
$$3n^2 + 10n - 8000 \le 0$$
$$n \le 50$$

Largest possible value of $n = 50$

2. (a) $T_1 = 4$, $T_2 = 7$, $T_3 = 10$

$$d = 3$$
$$T_{20} = 1 + 20(3)$$
$$= 61$$

(b) $T_1 + T_2 + T_3 + \ldots + T_{20}$

$$= \frac{20}{2}(4 + 61)$$
$$= 650$$

3. (a) $a = 18$, $d = 3$

$$S(n) \le 999$$

$$\frac{n}{2}[2(18)+(n-1)(3)] \le 999$$

$$3n^2 + 33n \le 1998$$

$$n^2 + 11n - 666 \le 0$$

$$n \le 20.887$$

$\therefore$ maximum number of rows = 20

(b)

$$S(n) \ge 888$$

$$\frac{n}{2}[2(18)+(n-1)(3)] \ge 888$$

$$3n^2 + 33n \ge 1776$$

$$n^2 + 11n - 592 \ge 0$$

$$n \ge 19.445$$

$\therefore$ 20^{th} row