4 CENTRE OF TRIANGLE

Form 5
Vol 2

Part D – 4 Centre: Circumcentre

2. (a) Let r cm be the radius of the circumcircle.

$$\frac{10}{\sin 30^\circ} = 2r$$

$$r = 10\text{ cm}$$

(b) Draw a line from circumcenter O to AC and form point M, where $OM \perp AC$

$\because$ M is the mid - point of AC.

$\therefore AM = 5\text{ cm}$

Since $AO = 10\text{ cm}$, $AM = 5\text{ cm}$

The required perpendicular distance $\sqrt{10^2 - 5^2} = 5\sqrt{3}$ cm

4. Let the circumcenter of ΔOAB be $K(3, y)$

Mid-point of OA: $D(3,0)$

Mid-point of AB: $E\left(5, \frac{5}{2}\right)$

$\because KE \perp AB$

$\therefore$ Slope of $KE \times$ Slope of $AB = -1$

$$\left(\frac{y - \frac{5}{2}}{3-5}\right) \times \left(\frac{0-5}{6-4}\right) = -1$$

$$y = \frac{17}{10}$$

The coordinate of the circumcenter of ΔOAB : $\left(3, \frac{17}{10}\right)$

5. Mid-point of AO: $\left(0, \frac{5}{2}\right)$

Mid-point of AB: $(-2, 5)$

The coordinates of the circumcenter of ΔOAB : $\left(-2, \frac{5}{2}\right)$

8. Since $\angle AOB = 90^\circ$

Thus, the circumcentre is the mid-point of AB.

The coordinates of circumcentre are $\left(\frac{a}{2}, \frac{b}{2}\right)$.

$$2\left(\frac{a}{2}\right) - \left(\frac{b}{2}\right) + a = 0$$

$$4a = b$$

Area of ΔAOB

$= \frac{1}{2}ab$

$= \frac{1}{2}a(4a)$

$= 2a^2$

9. Slope of $AB = \frac{2-8}{-7-1} = \frac{-6}{-8} = \frac{3}{4}$

Slope of $BC = \frac{-8-8}{13-1} = \frac{-16}{12} = -\frac{4}{3}$

Slope of $AB \times$ slope of $BC = -1$

$\therefore AB \perp BC$

Circumcentre $= \left(\frac{-7+13}{2}, \frac{2-8}{2}\right) = (3, -3)$

10. $m_{L_1} = k$

$m_{L_2} = -\frac{1}{k}$

$\therefore m_{L_1} \times m_{L_2} = -1$

$L_1 \perp L_2$ and $\angle ACB = 90^\circ$

Circumcentre is the midpoint of AB
y-coordinate of circumcentre $= 0$
y-coordinate of $A = -7$

Put $C(0,c)$ into L_1, $c = 9$

Put $C(0,9)$ into L_2, $k = 4$

Put $y = -7$ into L_1, $A(-4,-7)$

Put $y = 7$ into L_2, $B(8,7)$

Thus, x-coordinate of circumcentre $= \frac{-4+8}{2} = 2$

12. Note that $y = k$ is the $\perp$ bisector of AC.
Thus, the coordinates of circumcentre are $(-2,k)$

$$(-2+5)^2 + (k-7)^2 = (-2-6)^2 + (k-2)^2$$
$$k = -1$$

$\therefore C(-5,-9)$

13. $\sqrt{(2-1)^2 + (7.5-2)^2} = \sqrt{(7-2)^2 + (k-7.5)^2}$

$1^2 + (5.5)^2 = 5^2 + (k-7.5)^2$

$(k-7.5)^2 = 6.25$

$k = 2.5 + 7.5$ or $-2.5 + 7.5$
$k = 10$ or 5

14. Since OA is horizontal

The coordinates of circumcentre are $\left(\frac{a}{2},12\right)$

Put $B(b,6)$ into $ax-8y-168=0$

$ab-48-168=0$

$ab=216$

$\left(\frac{a}{2}\right)^2+12^2=\left(\frac{a}{2}-b\right)^2+(12-6)^2$

$144=-ab+b^2+36$

$144=-216+b^2+36$

$b^2=324$

$b=18$

$a=12$

15. Assume circumcentre $(10,0)$

$(k-10)^2+(3-k)^2=(2k-10)^2+(k-12)^2$

$3k^2-38k+135=0$

$\Delta=(38)^2-4(3)(135)=-176<0$

Thus, it is not possible.

16. $A(12,0)$, $B(0,8)$

Let the coordinates of circumcentre be $(4,k)$.

$(12-4)^2+(0-k)^2=(0-4)^2+(8-k)^2$

$k=1$

The coordinates of circumcentre are $(4,1)$.

Let $C\left(h,\frac{11h+24}{3}\right)$

$(h-4)^2+\left(\frac{11h+24}{3}-1\right)^2=(0-4)^2+(8-1)^2$

$h=-3$

$\therefore C(-3,-3)$