4 CENTRE OF TRIANGLE

Form 5
Vol 2

Part C – 4 Centre: Orthocentre

1. Since $\angle OAB = 90^\circ$
   Thus, the coordinates of orthocentre are $(-4,0)$.

2. Let the coordinate of the orthocenter of ΔOAB be $C:(x,1)$

   Slope of AB: $\dfrac{1-3}{4-0} = -\dfrac{2}{4} = -\dfrac{1}{2}$

   Slope of OC: $\dfrac{1-0}{x-0} = \dfrac{1}{x}$

   $\because OC \perp AB$

   $\therefore \left(\dfrac{1}{x}\right)\times\left(-\dfrac{1}{2}\right) = -1$

   $x = \dfrac{1}{2}$

   The required orthocenter of ΔOAB : $\left(\dfrac{1}{2},1\right)$

3. Let the coordinate of the orthocenter of ΔOAB be $K(x, y)$

Slope of AO: $\frac{3-0}{1-0}=3$

Slope of BO: $\frac{2-0}{-1-0}=-2$

Slope of AK: $\frac{y-3}{x-1}$

Slope of BK: $\frac{y-2}{x+1}$

$\because AK \perp BO, BK \perp AO$

$\therefore$ Slope of $AK \times$ Slope of $BO=-1$, Slope of $BK \times$ Slope of $AO=-1$

$\begin{cases} x-2y+5=0 \\ x+3y-5=0 \end{cases}$

$x=-1, y=2$

The coordinate of the orthocenter of ΔOAB is $(-1,2)$

4. Let the coordinate of C be $(1, y)$ and I be the orthocenter.

$\because BI \perp AC$

$\therefore \left(\frac{3-\frac{21}{5}}{-5-1}\right)\left(\frac{y-3}{1-2}\right)=-1$

$y=8$

The coordinate of C: (1,8)

5. Put $y=0$ and $x=0$, we have $A\left(\frac{p}{3}, 0\right)$ and $B\left(0, \frac{p}{4}\right)$

Since BC is vertical, so OA is an altitude.

$\because AK=OK$, $\therefore K\left(\frac{p}{6}, 0\right)$

$\because BK \perp AC$

Slope of $AC=-1 \div \frac{\frac{p}{4}}{-\frac{p}{6}}=\frac{2}{3}$

Equation of AC is $y=\frac{2}{3}x+8$.

6. Coordinates of $C(-a,b)$

Note that AC is horizontal.

Thus, x-coordinate of $B=-4$, i.e. $b=-4$

Slope of $AB=\dfrac{-4-a}{a+4}=-1$

$$\frac{-4+1}{-a+4}\times -1=-1$$

$$a=7$$

$\therefore A(7,-4)$

7. Since OA is horizontal

Thus, x-coordinate of $K=-a$

Put $x=-a$, then $K\left(-a,\dfrac{-3a-b}{2}\right)$

Slope of $AB=\dfrac{-a}{2a+a}=-\dfrac{1}{3}$

$$\frac{\dfrac{-3a-b}{2}}{-a}\times -\frac{1}{3}=-1$$

$$3a=b$$

$\therefore a:b=1:3$

8. Since OA is horizontal

Thus, x-coordinate of $K=b$

Put $x=b$, then $K\left(b,\dfrac{b}{4}\right)$

$$\frac{\dfrac{b}{4}}{b}\times\frac{a}{b-a}=-1$$

$$3a=4b$$

$\therefore a:b=4:3$

9. (a) $f(x) = x^2 - 2ax + a^2 - 1$

$f(x) = (x-a)^2 - 1$

Vertex $(a, -1)$

(b) $A(a, -1)$, $B(a+13, -1)$

Assume orthocentre is $(3,1)$

Then x-coordinate of $C = 3$

Put $x = 3$, then we have $C(3,21)$.

$$\frac{-1-21}{a+13-3} \times \frac{-1-1}{a-3} = -1$$

$$a^2 + 7a + 14 = 0$$

$\Delta = 7^2 - 4(14) = -7 < 0$

Thus, the claim is disagreed.