4 CENTRE OF TRIANGLE

Form 5
Vol 2

Part A – Position in Different Triangle

1.

Center	Triangle	Inside Triangle (1)	On Triangle (2)	Outside Triangle (3)
Incenter	Obtuse angled triangle	√		
Circumcenter	Right angled triangle		√	
Orthocenter	Acute angled triangle	√		
Centroid	Obtuse angled triangle	√		
Incenter	Obtuse angled triangle	√		
Circumcenter	Right angled triangle		√	
Orthocenter	Obtuse angled triangle			√
Centroid	Acute angled triangle	√		

2. Let M be the mid-point of BC.
Then, $BM = 2.5\,\text{cm}$ and $AM \perp BC$.

$$AM = \sqrt{5^2 - (2.5)^2} = \frac{5\sqrt{3}}{2}\text{ cm}$$

$$AI = \frac{2}{3} \times AM = \frac{5\sqrt{3}}{3}\text{ cm}$$

3. Let M be the mid-point of BC.
Then, $BM = 6\,\text{cm}$ and $AM \perp BC$.

$$AM = \sqrt{10^2 - 6^2} = 8\text{ cm}$$

$$AK = \frac{2}{3} \times 8 = \frac{16}{3}\text{ cm}$$

4. (a) $\angle ABN = \angle CBM$ (*common* $\angle$)

$\angle ANB = \angle CMB = 90°$ (*Given*)

$\therefore \Delta ABN \sim \Delta CBM$ (*A.A.A*)

$BN = NC = 5cm$ (prop. of isos.Δ)

$\frac{AB}{CB} = \frac{BN}{BM}$ (corr. sides,~Δ)

$BM = \frac{50}{13}$ cm

(b) $\angle BAN = \angle MAY$ (*common* $\angle$)

$\angle ANB = \angle AMY = 90°$ (*Given*)

$\therefore \Delta ANB \sim \Delta AMY$ (A.A.A)

$AM = AB - BM = 13 - \frac{50}{13} = \frac{119}{13}$ cm

$AN = \sqrt{13^2 - 5^2} = 12cm$

$\frac{AN}{AM} = \frac{AB}{AY}$ (corr. sides, ~ Δ)

$AY = \frac{\left(\frac{119}{13}\right)(13)}{12} = \frac{119}{12}cm$

5. Denote the midpoint of the hypotenuse of ΔABC by M.
Note that $OIKM$ is a straight line.

$OI : OM = \sqrt{2} : \sqrt{2} + 1$

$OK : OM = 2 : 3$

$\frac{OI}{OK} = \frac{\sqrt{2}}{\sqrt{2}+1} \times \frac{3}{2}$

$\frac{OI}{OK} = \frac{3\sqrt{2}}{2\sqrt{2}+2}$

$\frac{OI}{IK} = \frac{3\sqrt{2}}{2\sqrt{2}+2-3\sqrt{2}}$

$= \frac{3\sqrt{2}}{2-\sqrt{2}}$

$= 3\sqrt{2} + 3$

Part B – 4 Centre: Centroid

1. A

Draw a median from C to AB and form point G.

Then, the figure is formed by six triangles $\Delta AFG, \Delta BFG, \Delta BFD, \Delta CFD, \Delta CFE, \Delta AFE.$

Point F is the centroid of ΔABC.

Thus, Area of $\Delta AFG, \Delta BFG, \Delta BFD, \Delta CFD, \Delta CFE, \Delta AFE$ are same.

Hence, the ratio of the area ΔABF to quadrilateral $CEFD$: $\dfrac{\text{area of } \Delta AFG + \text{area of } \Delta BFG}{\text{area of } \Delta CFD + \text{area of } \Delta CFE} = 1:1$

2. Let G be the centroid of ΔABC and the coordinate of G be (x, y)

Let D be the mid-point of BC.

The coordinate of D: $(4,3)$

$\because$ The ratio of $AG:GD = 2:1$

$\therefore x = \dfrac{2\times4+1\times3}{2+1}, y = \dfrac{2\times3+1\times8}{2+1}$

$x = \dfrac{11}{3}, y = \dfrac{14}{3}$

The coordinate of G: $\left(\dfrac{11}{3}, \dfrac{14}{3}\right)$

3. Let the coordinate of C be (x, y), M be the mid-point of BC and K be the centroid of ΔABC.

$M: \left(\dfrac{7+x}{2}, \dfrac{6+y}{2}\right)$

$\because$ The ratio of $AK:KM = 2:1$

$\therefore \dfrac{\frac{(7+x)}{2}\times2+1}{3} = 6, \dfrac{\frac{(6+y)}{2}\times2+2\times1}{3} = 3$

$x = 10, y = 1$

The coordinate of C: $(10,1)$