

AS & GS

Form 6

Vol 1

Part 1 – Basic Concept

2. A

3. D

4. D

5. (a) $T(3)+T(13)$
 $=17-5(3)+17-5(13)$
 $=-46$

(b) $T(n)=-98$
 $17-5n=-98$
 $5n=115$
 $n=23$

(c) $T(n)=-303$
 $17-5n=-303$
 $5n=320$
 $n=64$
 \therefore yes, 64th term

6. (a) $T(n) > 0$
 $6n - 333 > 0$
 $6n > 333$
 $n > 55.5$
 \therefore The least value of $n = 56$.

(b) $T(n) = 38$
 $6n - 333 = 38$
 $6n = 371$
 $n = \frac{371}{6}$

Since n is an integer,

\therefore 38 is not a term in the sequence.

7. For $n = 3$,

$$\begin{aligned} a_5 &= a_4 + a_3 \\ &= 17 + 10 \\ &= 27 \end{aligned}$$

For $n = 4$,

$$\begin{aligned} a_6 &= a_5 + a_4 \\ &= 27 + 17 \\ &= 44 \end{aligned}$$

For $n = 5$,

$$\begin{aligned} a_7 &= a_6 + a_5 \\ &= 44 + 27 \\ &= 71 \end{aligned}$$

For $n = 6$,

$$\begin{aligned} a_8 &= a_7 + a_6 \\ &= 71 + 44 \\ &= 115 \end{aligned}$$

For $n = 7$,

$$\begin{aligned} a_9 &= a_8 + a_7 \\ &= 115 + 71 \\ &= 186 \end{aligned}$$

8. For $n = 2$,

$$\begin{aligned} a_4 &= a_3 + a_2 \\ 44 &= a_3 + 8 \\ a_3 &= 36 \end{aligned}$$

For $n = 3$,

$$\begin{aligned} a_5 &= a_4 + a_3 \\ &= 44 + 36 \\ &= 80 \end{aligned}$$

For $n = 4$,

$$\begin{aligned} a_6 &= a_5 + a_4 \\ &= 80 + 44 \\ &= 124 \end{aligned}$$

For $n = 5$,

$$\begin{aligned} a_7 &= a_6 + a_5 \\ &= 124 + 80 \\ &= 204 \end{aligned}$$

For $n = 6$,

$$\begin{aligned} a_8 &= a_7 + a_6 \\ &= 204 + 124 \\ &= 328 \end{aligned}$$

Part 2 – Arithmetic Sequence

4. (a) $T(3) = 29$

$$a + 2d = 29 \dots (1)$$

$$T(6) = 41$$

$$a + 5d = 41 \dots (2)$$

$$\therefore \text{we have } d = 4, a = 21$$

$$\text{General term} = 4n + 17$$

(b) $T(n) + T(n+1) = 214$

$$4n + 17 + [4(n+1) + 17] = 214$$

$$8n = 176$$

$$n = 22$$

\therefore 22nd term and 23rd term

5. (a) $T(2) = 113$

$$a + d = 113 \dots (1)$$

$$T(10) = 73$$

$$a + 9d = 73 \dots (2)$$

$$\therefore \text{we have } d = -5, a = 118$$

$$\text{General term} = -5n + 123$$

(b) $T(n) = -32$

$$-5n + 123 = -32$$

$$5n = 155$$

$$n = 31$$

\therefore 31st term

(c) $T(k) < 0$

$$-5k + 123 < 0$$

$$k > 24.6$$

\therefore 25th term

$$6. \quad (a) \quad T(6) + T(11) = 86$$

$$a + 5d + a + 10d = 86$$

$$2a + 15d = 86 \dots (1)$$

$$T(23) = -15$$

$$a + 22d = -15 \dots (2)$$

$$\therefore \text{ we have } d = -4, \quad a = 73$$

$$\text{General term} = -4n + 77$$

$$(b) \quad T(n) = 1$$

$$-4n + 77 = 1$$

$$4n = 76$$

$$n = 19$$

\therefore yes, 19th term

$$7. \quad T(n) = (n+1)(n+2) - (n+3)(n+4)$$

$$T(1) = (2)(3) - (4)(5) = -14$$

$$T(2) = (3)(4) - (5)(6) = -18$$

$$T(3) = (4)(5) - (6)(7) = -22$$

$$T(2) - T(1) = -4$$

$$T(3) - T(2) = -4$$

$$T(2) - T(1) = T(3) - T(2)$$

\therefore AS

$$8. \quad (a) \quad a = -2, \quad d = 0 - (-2) = 2, \quad T(n) = 244$$

$$-4 + 2n = 244$$

$$n = 124$$

\therefore 124 terms

$$(b) \quad a = 13, \quad d = 4 - 13 = -9, \quad T(n) = -149$$

$$22 - 9n = -149$$

$$n = 19$$

\therefore 19 terms

$$(c) \quad (4a + b) + (2a - 3b) = 2(3b)$$

$$6a - 2b = 6b$$

$$6a = 8b$$

$$3a = 4b$$

first term $= 2a - 3b, \quad d = -2a + 6b$

$$(2a - 3b) + (n - 1)(-2a + 6b) = 3a + 29b$$

$$(n - 1)(-2a + 6b) = a + 32b$$

$$(n - 1)\left[-2\left(\frac{4b}{3}\right) + 6b\right] = \frac{4b}{3} + 32b$$

$$(n - 1)\left(\frac{10b}{3}\right) = \frac{100b}{3}$$

$$n = 11$$

\therefore 11 terms

$$9. \quad a = 3001, \quad d = 2668 - 3001 = -333$$

$$T(n) > 0$$

$$3334 - 333n > 0$$

$$n < 10\frac{4}{333}$$

\therefore 10 terms

$$10. \quad a = 277, \quad d = 254 - 277 = -23$$

$$T(n) > 0$$

$$300 - 23n > 0$$

$$n < 13\frac{1}{23}$$

\therefore The smallest term

$$= T(13)$$

$$= 300 - 23(13)$$

$$= 1$$

Part 3 - Properties of A.S.

3. (a) $T(1) = 8$
 $a = 8$

$$T(6) = 20$$
$$8 + (6-1)d = 20$$
$$d = \frac{12}{5}$$

$$\therefore \text{common difference} = \frac{12}{5}$$

(b) $T(3) = b$

$$b = 8 + (3-1)\left(\frac{12}{5}\right)$$
$$= \frac{64}{5}$$
$$b^2 = \frac{4096}{25}$$

4. (a) $-2x + 3x + 2 = 2(-x + 4)$
 $x + 2 = -2x + 8$
 $3x = 6$
 $x = 2$

(b) $T(1) = 3x + 2 = 8$
 $T(2) = -x + 4 = 2$
 $T(3) = -2x = -4$
 $\therefore \text{general term} = 14 - 6n$

5. sum of roots

$$\alpha + \beta = \frac{3(3k+11)}{2}$$
$$16 = \frac{3(3k+11)}{2}$$
$$k = -\frac{1}{9}$$