COORDINATE GEOMETRY(III)

Form 6

Vol 6

Part 5 – 4 centre questions

18.

Let the coordinate of incenter be $I(x, y)$

length of $AB = 25$ units, length of $BO = 25$ units and length of $OC = 30$ units

Draw three lines from I to AO, OB, AB and form point C, D, E respectively.

Where $IC \perp AO$, $ID \perp BO$, $IE \perp AB$

$\because \Delta AOB$ is an isosceles triangle

$BC \perp AO$ and $AO = OC = 15$ units (prop. of isos. Δ)

$OC = OD = 15$ units, $AC = AE = 15$ units, $BD = BE = 25 - 15 = 10$ units (tangent prop.)

Thus, the x-coordinate of I is 15.

Since $IC \perp AO$,

slope of IC $\times$ slope of AO $= -1$

Coordinate of point C:(9.12)

$$\left(\frac{y-12}{15-9}\right) \times \frac{24-0}{18-0} = -1$$

$$y = \frac{15}{2}$$

The coordinate of I: $\left(15, \frac{15}{2}\right)$

19.

Let the coordinate of C be $(1, y)$ and I be the orthocenter.

$\because$ BI $\perp$ AC

$\therefore \left(\frac{3-\frac{21}{5}}{-5-1}\right)\left(\frac{y-3}{1-2}\right) = -1$

$y = 8$

The coordinate of C: (1,8)

20. Note that $A(a,0)$ lies on $x+5y=a$.

Let the inclination of straight line passing through AB be θ.

$$-\tan\left(\frac{180^\circ-\theta}{2}\right)=-\frac{1}{5}$$

$$\tan\left(90^\circ-\frac{\theta}{2}\right)=\frac{1}{5}$$

$$\tan\frac{\theta}{2}=5$$

$$\tan\theta=-\frac{5}{12}$$

$$-\frac{b}{a}=-\frac{5}{12}$$

$\therefore a:b=12:5$

21. Note that AB is horizontal and $\angle BAO=90^\circ$.

$$OB=\sqrt{3^2+a^2}$$

$$(a-1)+2=\sqrt{3^2+a^2}$$

$$(a+1)^2=9+a^2$$

$$2a+1=9$$

$$a=4$$

22. Slope of $AB = \frac{18+6}{-6+13} = \frac{24}{7}$

Slope of $BC = -\frac{4}{3}$

Let the inclination of AB and BC be α and β respectively.

$\tan\alpha = \frac{24}{5}$ and $\tan\beta = -\frac{4}{3}$

Slope of the angle bisector of $\angle ABC = \tan\left(\frac{\alpha+\beta}{2}\right) = -\frac{11}{2}$

$$\begin{cases} 3x-4y=15 \\ y-18=-\frac{11}{2}(x+6) \end{cases}$$

Thus, the incentre $(-1.8,-5.1)$.

23. y-int of $L_1 = 2$

Note that the triangle formed is isos. with the axis of symmetry $x=0$.

Thus, incentre $(0,15)$

$$\begin{cases} y=a \\ 12x-5y+10=0 \end{cases}$$

Solving, we have $\left(\frac{5a-10}{12}, a\right)$

Let θ be the inclination of L_1.

$\tan\theta = \frac{12}{5}$

$$\frac{a-15}{\frac{5a-10}{12}-0} = \tan\frac{\theta}{2}$$

$$12\left(\frac{a-15}{5a-10}\right) = \frac{2}{3}$$

$$18a-270 = 5a-10$$

$$13a = 260$$

$$a = 20$$

24. Note that $A(5,0)$ lies on $2x-3y-10=0$.

Let θ be the inclination of $2x-3y-10=0$

$$\tan\theta=\frac{2}{3}$$

$$\tan 2\theta=\frac{12}{5}$$

Thus, slope of $AB=\frac{12}{5}$

Let $B(0,b)$

$$\frac{b}{-5}=\frac{12}{5}$$

$$b=-12$$

$\therefore B(0,-12)$