COORDINATE GEOMETRY(III)

Form 6
Vol 6

Part 4 – Shortest/farthest distance

1.

Centre $= (8,5)$, radius $= \sqrt{8^2+5^2+32} = 11$

Shortest distance from centre to L

$$= \left|\frac{13(8)+10(5)+115}{\sqrt{13^2+10^2}}\right| = \sqrt{269}$$

Shortest distance from P to $L = \sqrt{269} - 11$

Part 5 – 4 centre questions

1.

Center	Triangle	Inside Triangle (1)	On Triangle (2)	Outside Triangle (3)
Incenter	Obtuse angled triangle	√		
Circumcenter	Right angled triangle		√	
Orthocenter	Acute angled triangle	√		
Centroid	Obtuse angled triangle	√		
Incenter	Obtuse angled triangle	√		
Circumcenter	Right angled triangle		√	
Orthocenter	Obtuse angled triangle			√
Centroid	Acute angled triangle	√		

2. (a) T (False when ΔABC is right)
   (b) T (False when ΔABC is right)
   (c) T
   (d) T (False when ΔABC is right)
   (e) T (False when ΔABC is right)
   (f) T (False when ΔABC is right)

3.

Let the coordinate of the orthocenter of ΔOAB be $C: (x, 1)$

Slope of AB: $\frac{1-3}{4-0} = -\frac{2}{4} = -\frac{1}{2}$

Slope of OC: $\frac{1-0}{x-0} = \frac{1}{x}$

$\because OC \perp AB$

$\therefore \left(\frac{1}{x}\right) \times \left(-\frac{1}{2}\right) = -1$

$x = \frac{1}{2}$

The required orthocenter of ΔOAB: $(\frac{1}{2}, 1)$

5. (a) $C(4, -5)$

(b) Note that AC is vertical.

Let $K(h, 7)$

$AK \perp BC$

$$\frac{7-9}{h-4} \times \frac{7+5}{16-4} = -1$$

$$h - 4 = 2$$

$$h = 6$$

$\therefore K(6, 7)$

6. (a) $f(x)$

$= \frac{1}{4}x^2 - 4x + 10$

$= \frac{1}{4}(x^2 - 16x + 8^2 - 8^2) + 10$

$= \frac{1}{4}(x-8)^2 - 6$

$\therefore A(8, -6)$

(b) $B(8, k-6)$

(c) Note that AB is vertical and $C(0,10)$.

Let $P(h, 10)$

$AP \perp BC$

$\frac{10+6}{h-8} \times \frac{k-6-10}{8} = -1$

$h = 40 - 2k$

Put $P(40-2k, 10)$ into $y = f(x) + k$

$10 = \frac{1}{4}(40 - 2k - 8)^2 - 6 + k$

$k^2 - 31k + 240 = 0$

$k = 15$ or $k = 16$

Thus, when $k = 15$ or $k = 16$, P lies on $y = f(x) + k$.

Yes, it is possible.

7. Put $A(a, a)$ into $L: x + 3y - b = 0$

$a + 3a - b = 0$

$b = 4a$

Put $y = 0$ into L, then $B(b, 0)$

Since B is the orthocentre, then $AB \perp BC$

Let $C(a, c)$

$\frac{c}{a-b} \times \frac{a}{a-b} = -1$

$c = -9a$

$\therefore C(a, -9a)$

$\frac{a - 9a}{2} = -4$

$a = 1$

The area of $\Delta ABC = \frac{1}{2}(1+9)(4-1) = 15$

8. Note that $\angle AOB = 90^\circ$.
The coordinates of the circumcentre
= the coordinates of midpoint of AB
$= (10, -4)$

9.
Let the circumcentre be (h, k).

$$(h-2)^2+(k-5)^2=(h-8)^2+(k-11)^2=(h+4)^2+(k-1)^2$$
$$-4h+4-10k+25=-16h+64-22k+121=8h+16-2k+1$$
$$\begin{cases}-4h+4-10k+25=-16h+64-22k+121\\-16h+64-22k+121=8h+16-2k+1\end{cases}$$
$$\begin{cases}12h+12k-156=0\\24h+20k-168=0\end{cases}$$
$$\begin{cases}h=-23\\k=36\end{cases}$$

Circumcentre $(-23, 36)$

11. Note that AC is horizontal.
Let the circumcentre be $(-4, k)$

$$\sqrt{(-13+4)^2+(-2-k)^2}=\sqrt{(-1+4)^2+(10-k)^2}$$
$$81+4k+4=9-20k+100$$
$$24k=24$$
$$k=1$$

Circumcentre $(-4, 1)$

13.

Perpendicular bisector of AB

$(x-2)^2+(y-3)^2=(x-5)^2+(y-7)^2$

$6x+8y-61=0$

$$\begin{cases}5x+y-3=0\\6x+8y-61=0\end{cases}$$

Solving, we have circumcentre $\left(-\frac{37}{34},\frac{287}{34}\right)$

Let $C(x,y)$

$$\left(x+\frac{37}{34}\right)^2+\left(y-\frac{287}{34}\right)^2=\left(2+\frac{37}{34}\right)^2+\left(3-\frac{287}{34}\right)^2$$

$$x^2+y^2+\frac{37}{17}x-\frac{287}{17}y=4+9+\frac{74}{17}-\frac{861}{17}$$

$17x^2+17y^2+37x-287y+566=0$

14. Put $A(a,k)$ into $y=\frac{1}{2}x$, then $a=2k$.

Put $B(b,k)$ into $y=-\frac{4}{5}x$, then $b=-\frac{5}{4}k$.

Note that AB is horizontal.

Let the circumcentre $K\left(\frac{a+b}{2},c\right)$, i.e. $K\left(\frac{3}{8}k,c\right)$

Put $K\left(\frac{3}{8}k,c\right)$ into $6x-7y+p=0$

$6\left(\frac{3}{8}k\right)-7c+p=0$

$9k-28c+4p=0$

$c=\frac{9k+4p}{28}$

$K\left(\frac{3}{8}k,\frac{9k+4p}{28}\right)$

$KO=KA$

$$\left(\frac{3}{8}k\right)^2+\left(\frac{9k+4p}{28}\right)^2=\left(\frac{3}{8}k-a\right)^2+\left(\frac{9k+4p}{28}-k\right)^2$$

$$-\frac{3}{4}ka+a^2-\left(\frac{9k+4p}{14}\right)k+k^2=0$$

$$-\frac{3}{2}k^2+4k^2-\left(\frac{9k+4p}{14}\right)k+k^2=0$$

$$\frac{7}{2}k-\frac{9k+4p}{14}=0$$

$$49k-9k-4p=0$$

$$40k=4p$$

$\therefore p:k=10:1$

15. Slope of $L_1=1$

Slope of $AB=-1$

$$\begin{cases}x-y+10=0\\y+5=-(x+5)\end{cases}$$

Thus, we have midpoint of AB is $(-10,0)$.

$\therefore B(-15,5)$

Slope of $L_2=3$

Slope of $BC=-\frac{1}{3}$

$$\begin{cases}3x-y+10=0\\y-5=-\frac{1}{3}(x+15)\end{cases}$$

Thus, we have midpoint of BC is $(-3,1)$.

$\therefore C(9,-3)$

$$(x+5)^2+(y+5)^2=(x-9)^2+(y+3)^2$$

$$10x+25+10y+25=-18x+81+6y+9$$

$$7x+y-10=0$$

Equation of perpendicular bisector of AC: $7x+y-10=0$.

17. $A(12,0)$ and $B(0,8)$

Let the circumcentre be $K(4,k)$

$$(12-4)^2+(0-k)^2=(0-4)^2+(8-k)^2$$

$$64=16-16k+64$$

$$k=1$$

$\therefore K(4,1)$

Let $C\left(c,\dfrac{11c+24}{3}\right)$

$$(c-4)^2+\left(\frac{11c+24}{3}-1\right)^2=(0-4)^2+(8-1)^2$$

$$x(x+3)=0$$

$x=-3$ or $x=0$

$\therefore C(-3,-3)$