COORDINATE GEOMETRY(III)

Form 6
Vol 6

Part 3 – Intersection of straight line and circle

4.

$$\begin{cases} y = 3x - 40 \\ x^2 + y^2 - 18x - ky + 50 = 0 \end{cases}$$

$$\begin{aligned} x^2 + (3x-40)^2 - 18x - k(3x-40) + 50 &= 0 \\ 10x^2 - (3k+258)x + 1650 + 40k &= 0 \end{aligned}$$

x-coordinate of mid-point $= \dfrac{3k+258}{10(2)} = \dfrac{3k+258}{20}$

y-coordinate of mid-point $= 3\left(\dfrac{3k+258}{20}\right) - 40 = \dfrac{9k-26}{20}$

$\therefore$ mid-point $= \left(\dfrac{3k+258}{20}, \dfrac{9k-26}{20}\right)$

5.

Let $(x, \dfrac{3x+29}{4})$ be the centre.

$$\begin{aligned} \sqrt{x^2 + (\frac{3x+29}{4} - 9)^2} &= \sqrt{x^2 + (\frac{3x+29}{4} - 1)^2} \\ x^2 + (\frac{3x-7}{4})^2 &= x^2 + (\frac{3x+25}{4})^2 \\ 9x^2 - 42x + 49 &= 9x^2 + 150x + 625 \\ x &= -3 \end{aligned}$$

$\therefore$ Centre $= (-3, 5)$

Radius $= \sqrt{(-3)^2 + (5-9)^2} = 5$

Equation of C: $(x+3)^2 + (y-5)^2 = 25$

6.

Let $y = 4x + c$ be the equation of tangent.

$$x^2 + (4x+c)^2 - 6x + 2(4x+c) - 7 = 0$$
$$17x^2 + (8c+2)x + (c^2 + 2c - 7) = 0$$

$$\Delta = 0$$
$$(8c+2)^2 - 4(17)(c^2 + 2c - 7) = 0$$
$$-4c^2 - 104c + 480 = 0$$
$$c = -30 \text{ or } c = 4$$

Equation of tangents: $y = 4x - 30$ and $y = 4x + 4$

7.

Let $y = -x + c$ be the equation of tangent.

$$x^2 + (-x+c)^2 + 10x - 4(-x+c) - 3 = 0$$
$$2x^2 + (-2c+14)x + (c^2 - 4c - 3) = 0$$

$$\Delta = 0$$
$$(-2c+14)^2 - 4(2)(c^2 - 4c - 3) = 0$$
$$-4c^2 - 24c + 220 = 0$$
$$c = -11 \text{ or } c = 5$$

Equation of tangents: $y = -x - 11$ and $y = -x + 5$

8.

Centre $= (5, -4)$

Equation of tangent:

$$\frac{y}{x-4} = (-1) \div \left(\frac{-4-0}{5-4}\right) = \frac{1}{4}$$
$$y = \frac{1}{4}x - 1$$

9.

Let m be the slope of tangent.

Equation of tangent:

$$\frac{y+4}{x-3}=m$$

$$mx-y-(3m+4)=0$$

Note that the shortest distance from the centre to the tangent is the radius.

Centre $=(-1,4)$, Radius $=\sqrt{1^2+4^2-9}=\sqrt{8}$

By the shortest distance formula,

$$\left|\frac{-m-4-(3m+4)}{\sqrt{m^2+1}}\right|=\sqrt{8}$$

$$\frac{-m-4-(3m+4)}{\sqrt{m^2+1}}=\sqrt{8} \text{ or } \frac{-m-4-(3m+4)}{\sqrt{m^2+1}}=-\sqrt{8}$$

$$-4m-8=\sqrt{8m^2+8} \text{ or } -4m-8=-\sqrt{8m^2+8}$$

$$16m^2+64m+64=8m^2+8$$

$$8m^2+64m+56=0$$

$$m=-1 \text{ or } m=-7$$

$\therefore$ Equations of tangents: $y=-x-1$ and $y=-7x+17$

10.

(a) $x^2+y^2=144$

(b) $\sin\alpha=\dfrac{3}{\sqrt{3^2+4^2}}=\dfrac{3}{5}$

$$OQ=\frac{OP}{\sin\alpha}=\frac{12}{0.6}=20$$

11.

(a) (i) $Q = (0, 5)$

(ii) $(x+2)^2 + (y-5)^2 = 4$

(b) $\begin{cases} y = mx \\ (x+2)^2 + (y-5)^2 = 4 \end{cases}$

$(1+m^2)x^2 + (4-10m)x + 25 = 0$

$\Delta = 0$

$(4-10m)^2 - 4(1+m^2)(25) = 0$

$m = -\dfrac{21}{20}$

(c) (i) $\angle PQO = 90°$(tangent $\perp$ radius)

$\angle PRO = 90°$(tangent $\perp$ radius)

$\angle PQO + \angle PRO = 180°$

O, Q, P, R are concyclic (opp.$\angle$s supp.)

(ii) Diameter $= OP = \sqrt{29}$

Radius $= \dfrac{\sqrt{29}}{2}$

Centre $= (-1, \dfrac{5}{2})$

Equation of circle: $(x+1)^2 + (y - \dfrac{5}{2})^2 = \dfrac{29}{4}$

12.

(a) Centre $= (-3, 0)$

$\frac{-p}{2} = -3 \Rightarrow p = 6$

Put $x = -6, y = 0$

$0 = m(-6) + 6\sqrt{2}$

$m = \sqrt{2}$

(b) $\begin{cases} y = \sqrt{2}x + 6\sqrt{2} \\ x^2 + y^2 + 6x = 0 \end{cases}$

$$x^2 + (\sqrt{2}x + 6\sqrt{2})^2 + 6x = 0$$
$$x^2 + 2(x+6)^2 + 6x = 0$$
$$3x^2 + 30x + 72 = 0$$
$$x = -4 \text{ or } -6$$

When $x = -4, y = 2\sqrt{2}$

$\therefore B = (-4, 2\sqrt{2})$

(c) $P = (0, 6\sqrt{2})$

(d) Let $x^2 + y^2 + Dx + Ey + F = 0$ be the equation of C_2.

$$\begin{cases} -6D + F + 36 = 0 \\ -3D + F + 9 = 0 \\ 6\sqrt{2}E + F + 72 = 0 \end{cases}$$

(2) – (1),

$3D = 27$

$D = 9$

$F = -36 + 6(9) = 18$

$E = \frac{-72-18}{6\sqrt{2}} = \frac{-15\sqrt{2}}{2}$

Equation of C_2: $2x^2 + 2y^2 + 18x - 15\sqrt{2}y + 36 = 0$

(e) Centre of $C_2\left(-\frac{9}{2},\frac{15\sqrt{2}}{4}\right)$

Slope of $L_2=(-1)\div\left(\dfrac{\frac{15\sqrt{2}}{4}-6\sqrt{2}}{-\frac{9}{2}}\right)=-\sqrt{2}$

Equation of L_2: $y=-\sqrt{2}x+6\sqrt{2}$

13.

(a) Mid-point of $OA=(\frac{0+4}{2},\frac{0+16}{2})=(2,8)$

Equation of L:

$$\frac{y-8}{x-2}=(-1)\div(\frac{16}{4})=-\frac{1}{4}$$
$$x+4y-34=0$$

(b) (i) Note that L passes through M.

When $x=h, y=k$

$$h+4k-34=0$$
$$h=34-4k$$

(ii) Let $P=(x,y)$

$$PM=OM$$
$$\sqrt{(x-34+4k)^2+(y-k)^2}=\sqrt{h^2+k^2}$$
$$(x-34)^2+8k(x-34)+16k^2+y^2-2ky+k^2=h^2+k^2$$
$$x^2+y^2+8kx-68x-2ky+16k^2-272k-h^2+1156=0$$
$$x^2+y^2+4(2k-17)x-2ky+16k^2-272k-(34-4k)^2+1156=0$$
$$x^2+y^2+4(2k-17)x-2ky=0$$

(c) Centre $=M=(34-4k,k)$

$$\frac{16-k}{4-34+4k}\times\frac{2}{3}=-1$$
$$\frac{16-k}{4k-30}=-\frac{3}{2}$$
$$32-2k=-12k+90$$
$$k=\frac{29}{5}$$