REGULAR QUIZ 03

Form 6

Coordinate geometry (II), (III)

Part A – MC (@3 marks)

1.	(A)	This question is cancelled. The solution below is "reflected about x-axis" Let $f(x)=3^x$ After reflection, $-f(x)=-3^x$ After compression, $-f(2x)=-3^{2x}$ After translation, $-f(2(x+2))=-3^{2x+4}$
2.	C	Since $f(0)=-3$, $f(2)=f(4)=0$ Then $g(0)=-\frac{1}{2}f(0)=\frac{3}{2}$, $g(2)=-\frac{1}{2}f(2)=0$ and $g(4)=-\frac{1}{2}f(4)=0$ $\therefore g(x)$ passes through $\left(0,\frac{3}{2}\right)$, $(2,0)$ and $(4,0)$
3.	A	Maximum and minimum of the graph are 0 and -4 respectively and it attains its maximum when $x=40$. $\therefore y=2\sin\left(x^\circ+50^\circ\right)-2$
4.	B	Only II is incorrect. Its locus is perpendicular bisector of AE.
5.	B	Equation of P $AP=PB$ $(x-2)^2+(y+3)^2=(x+4)^2+(y-1)^2$ $-4x+4+6y+9=8x+16-2y+1$ $12x-8y+4=0$ $3x-2y+1=0$
6.	B	Note that $L_1//L_2$ with slope $-\frac{3}{4}$ y-int of $L_1=\frac{5}{4}$ y-int of locus of $P=\frac{13}{16}$ Equation of L_2

		$y=-\frac{3}{4}x+\frac{13}{16}+\left(\frac{13}{16}-\frac{5}{4}\right)$ $y=-\frac{3}{4}x+\frac{3}{8}$ $6x+8y-3=0$
7.	B	Slope of $AB=2$ y-int of $AB=-1$ Let L be a straight line parallel to AB and passes through C. Then y-int of L $\frac{y-2}{0-6}=2$ $y=-10$ $\therefore$ The locus of P $y=2x-1+(-1+10)$ $2x-y+8=0$
8.	A	$x^2+y^2-2x+6ky-37.5=0$ Centre $G(1,-3k)$ $r^2=1^2+(-3k)^2+37.5=9k^2+38.5$ $AG^2=(2k-1)^2+(3-2k+3k)^2=5k^2-10k+10$ Consider r^2-AG^2 $=4k^2+10k+28.5$ $=4\left(k+\frac{5}{4}\right)^2+\frac{89}{4}$ Since $\left(k+\frac{5}{4}\right)^2\geq 0$, $\therefore r^2-AG^2>0\Rightarrow r>AG$ A must lies inside C. I is true. If $MN=18$, $1^2+\left(\frac{18}{2}\right)^2=r^2$ $9k^2+38.5=82$ $k^2=\frac{29}{6}$ II is false. Minimum area of the circle $=38.5\pi$ III is false.

9.	D	Centre (h,k), radius $=k$ and $0<h<k$ $\therefore$ centre lies in quadrant 1 with x-axis is its tangent.
10.	A	Let centre be (h,r) $\sqrt{(h-3)^2+(r-1)^2}=r$ $(h-3)^2+r^2-2r+1=r^2$ $(h-3)^2+1=2r$ $(h-3)^2+1>26$ $(h-3)^2>25$ $h-3<-5$ or $h-3>5$ $h<-2$ $\quad$ $h>8$
11.	A	Perpendicular bisector of AB $(x-8)^2+(y-5)^2=(x-4)^2+(y+3)^2$ $x+2y-8=0$ $\begin{cases} x+2y-8=0 \\ 3x-4y+6=0 \end{cases}$ $\therefore$ Centre $(2,3)$ Equation of circle $(x-2)^2+(y-3)^2=(8-2)^2+(5-3)^2$ $x^2+y^2-4x-6y-27=0$
12.	C	Denote centre be $C(-1,2)$ and the mid-point of AB be M. Equation of MC $y-2=\frac{1}{3}(x+1)$ $x-3y+7=0$ $\begin{cases} 3x+y-k=0 \\ x-3y+7=0 \end{cases}$ $x=\frac{3k-7}{10}$
13.	A	$\left\lvert\frac{-1+3+m}{\sqrt{1^2+1^2}}\right\rvert\le\sqrt{8}$ $-4\le m+2\le 4$ $-6\le m\le 2$

1.	(A)	2.	C	3.	A	4.	B	5.	B	6.	B	7.	B
8.	A	9.	D	10.	A	11.	A	12.	C	13.	A		

Part B - Long Questions

1. (8 marks)

(a) $f(x)$

$=4x^2-12x+k+7$

$=4\left(x-\frac{3}{2}\right)^2+k-2$ 1M

$\therefore U\left(\frac{3}{2},k-2\right)$ 1A

(b)(i) $g(x)$

$=-x^2+6x-k-3$

$=-(x-3)^2-k+6$

$\therefore V(3,-k+6)$ 1A

$f(x)$ is reflected about the x-axis, translated upwards by 4 units 1A

and enlarged to 2 times along the x-axis +1A

(b)(ii) Note that $OU \perp UV$. 1M

$\frac{k-2}{\frac{3}{2}}\times\frac{k-2-(-k+6)}{\frac{3}{2}-3}=-1$ 1M

$8k^2-48k+55=0$

$k=\frac{48\pm\sqrt{48^2-4(8)(55)}}{2(8)}$

$k=3+\frac{\sqrt{34}}{4}$ or $k=3-\frac{\sqrt{34}}{4}$ (r.t. $k=4.46$ or $k=1.54$) 1A

2. (10 marks)

(a) Let the equation of C be $x^2+y^2+Dx+Ey+F=0$

Put $(0,0)$, we have $F=0$ — 1A

Put $A(2,4)$, $2D+4E+20=0$

Put $B(6,7)$, $6D+7E+85=0$

Solving, $D=-20$, $E=5$ — 1A for either one

The equation of C

$x^2+y^2-20x+5y=0$ — 1A

(b) Let the equation of tangent be $y-3=m(x-22)$

i.e. $y=mx-22m+3$

$x^2+(mx-22m+3)^2-20x+5(mx-22m+3)=0$ — 1M

$(1+m^2)x^2+(-44m^2+11m-20)x+484m^2-242m+24=0$

$\Delta=0$

$(-44m^2+11m-20)^2-4(1+m^2)(484m^2-242m+24)=0$ — 1M

$-151m^2+528m+304=0$

$m=4$ or $m=-\dfrac{76}{151}$

The equation of the tangents

$4x-y-85=0$ $\quad$ $76x+151y-2125=0$ — 1A+1A

(c) Denote $4x-y-85=0$ and $76x+151y-2125=0$ be L_1 and L_2

Inclination of L_1

$\tan\theta_1=4$

$\theta_1=75.96375653$

Inclination of L_2

$\tan\theta_2=-\dfrac{76}{151}$

$\theta_2=153.283373$

$\angle KVM=\dfrac{1}{4}(\theta_2-\theta_1)=19.3^\circ$

1M for $\theta_2-\theta_1$

or finding $\angle UKV=\theta_1+180^\circ-\theta_2$

alternatively

+1M for $\dfrac{1}{4}$

+1A

3. (12 marks)

(a) Let centre be $(4, r)$

$\sqrt{(4-0)^2+(r-8)^2}=r$

$r=5$ 1A

Equation of C

$(x-4)^2+(y-5)^2=25$ 1A

(b) Slope of $L_1=-\frac{3}{4}$ 1M

Equation of L_2

$y-5=\frac{4}{3}(x-4)$ 1M

$4x-3y-1=0$ 1A

(c)(i) $\begin{cases}3x+4y+18=0\\4x-3y-1=0\end{cases}$

$S(-2,-3)$ 1A

(c)(ii) Shortest distance

$=\sqrt{(4+2)^2+(5+3)^2}-5$ 1M for $\sqrt{(4+2)^2+(5+3)^2}$

+1M for -5

$=5$ 1A

(c)(iii) Centre of the required circle

$=\left(\frac{4+3(-2)}{1+3},\frac{5+3(-3)}{1+3}\right)$ 1M

$=\left(-\frac{1}{2},-1\right)$

Equation of the required circle

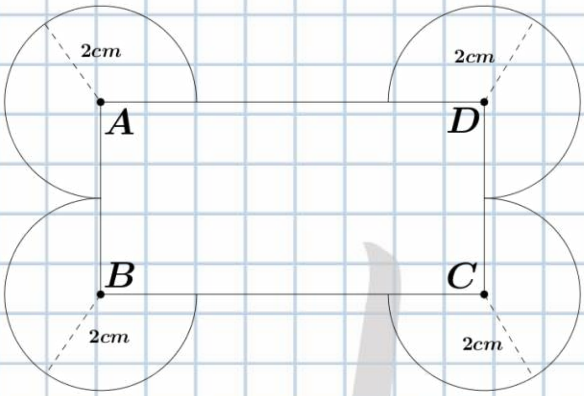
$\left(x+\frac{1}{2}\right)^2+(y+1)^2=\left(\frac{5}{2}\right)^2$ 1M for $\frac{5}{2}$

+1A

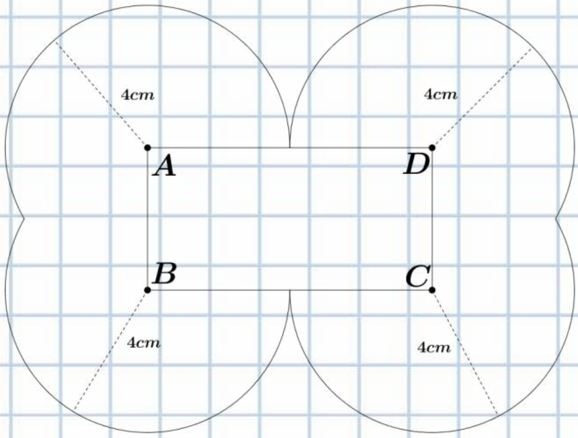
$x^2+y^2+x+2y-5=0$

4. (Bonus) (9marks)(3@)

(a)



(b)



(c)