COORDINATE GEOMETRY(III)

Form 6

Vol 6

Part 1 – Equation of circle

1. C
2. A
3. B
4.

(a) Put $x=0$,

$3y^2+42y+39=0$

$y=-13$ or $y=-1$

$P(0,-13)$, $Q(0,-1)$

Put $y=0$,

$3x^2-22x+39=0$

$x=3$ or $x=\frac{13}{3}$

$R(3,0)$, $S(\frac{13}{3},0)$

(b) Area $=\frac{(-1+13)\times 3}{2}=18$ sq. units

5.

Centre $(3,4)$

$B(3,3)$

Part 2 – Condition of equation of circle

1. D

2.

Let centre be $(h,0)$

$(h-5)^2+(0-10)^2=(h+1)^2+(0-8)^2$

$h=5$

Equation of circle

$(x-5)^2+y^2=100$

3. A

4.

(a) $(-3,7)$

(b) 3

(c) $\sqrt{3^2+7^2-k}=3$

$k=49$

5.

(a) Let centre be $C(r,r)$

$AP=AQ=8-r$

$BP=BR=6-r$

$AB=\sqrt{6^2+8^2}=10$

$8-r+6-r=10$

$r=2$

Centre $C(2,2)$

(b) $(x-2)^2+(y-2)^2=4$

6. D

7. B

8.

Radius $=\sqrt{3^2+\left(\frac{8}{2}\right)^2}=5$

Equation of C in general form

$(x-3)^2+(y-3)^2=5^2$

$x^2+y^2-6x-6y-7=0$

9.

ΔOAB is a right-angled triangle with $\angle AOB = 90^\circ$

Centre (3,1)

The required equation

$x^2 + y^2 - 6x - 2y = 0$

10.

Let $x^2 + y^2 + Dx + Ey + F = 0$ be the equation of circle.

$$\begin{cases} 13 - 3D + 2E + F = 0 \\ 29 + 2D + 5E + F = 0 \\ 52 + 4D - 6E + F = 0 \end{cases}$$

(2) – (1), we have $5D + 3E = -16$

(3) – (2), we have $2D - 11E = -23$

By solving, $D = -\frac{245}{61}, E = \frac{83}{61} \quad \therefore F = -\frac{1694}{61}$

Equation of circle:

$$x^2 + y^2 - \frac{245}{61}x + \frac{83}{61}y - \frac{1694}{61} = 0$$

$$61x^2 + 61y^2 - 245x + 83y - 1694 = 0$$

11.(a) Centre $= (-4,-4)$ Radius $= \sqrt{(-4)^2+(-4)^2-12} = 2\sqrt{5}$

(b) Put $x = 0$,

$y^2 + 8y + 12 = 0$

$y = -2$ or $y = -6$

$C = (0,-2), D = (0,-6)$

Put $y = 0$,

$x^2 + 8x + 12 = 0$

$x = -2$ or $x = -6$

$A = (-6,0), B = (-2,0)$

(c) Slope of AD $= \frac{-6-0}{0+6} = -1$

Slope of BC $= \dfrac{-2-0}{0+2} = -1$

$\therefore$ AD//BC

Slope of AK $= \dfrac{0+4}{-6+4} = -2$

Slope of KC $= \dfrac{-4+2}{-4-0} = \dfrac{1}{2}$

Slope of AK $\times$ Slope of KC $= -2 \times \left(\dfrac{1}{2}\right) = -1$

$\therefore$ AK $\perp$ KC

(d) Equation of CK : $y = \frac{1}{2}x - 2$

Equation of AD : $y = -x - 6$

$$\begin{cases} y = \dfrac{1}{2}x - 2 \ldots (1) \\ y = -x - 6 \ldots (2) \end{cases}$$

$(1) - (2)$: $0 = \dfrac{3}{2}x + 4$

$x = -\dfrac{8}{3}$

$y = -\dfrac{10}{3}$

$\therefore P = (-\dfrac{8}{3}, -\dfrac{10}{3})$

(e) Area of KABC $= \frac{\sqrt{20}\times\sqrt{20}}{2} + \frac{4\times 2}{2}$

$= 14$ sq. units

Part 3 – Intersection of straight line and circle

1.

$x = 2 - 2y$

$(2-2y)^2 + y^2 + 2(2-2y) - 6y - k = 0$

$5y^2 - 18y + 8 - k = 0$

$\Delta > 0$

$18^2 - 4(5)(8-k) > 0$

$k > -8.2$

2. A

3.

(a) $y = mx$

(b) $\begin{cases} y = mx \\ x^2 + y^2 + 6x - 2y + 5 = 0 \end{cases}$

$x^2 + (mx)^2 + 6x - 2(mx) + 5 = 0$

$(1+m^2)x^2 + (6-2m)x + 5 = 0$

$\Delta = 0$

$(6-2m)^2 - 4(1+m^2)(5) = 0$

$-16m^2 - 24m + 16 = 0$

$m = -2 \text{ or } m = \frac{1}{2}$