TRIGONOMETRY 3D

Form 6
Vol 3

Part 5 – Non Square Base

5. (a) $\because VA = VC = VB$

$\therefore \Delta VLA \cong \Delta VLC \cong \Delta VLB$

$LA = LC = LB$ (corr.sides, $\cong \Delta$s)

$\therefore L$ is the circumcentre of ΔABC

(b) Let X be a point on BC such that $AX \perp BC$

$\because$ L is the circumcentre of ΔABC

LX is a perpendicular bisector of BC

$\angle CAX = \sin^{-1}\dfrac{4}{12} = 19.4712°$

$\because LA = LC = LB$

$\therefore \angle LCA = 19.4712°$

$\angle CLX = 38.9424°$

Distance between L and $ABC = LX = \dfrac{CX}{\tan 38.9424°} = 4.9498 \approx 4.95\text{ cm}$

(c) Required angle $= \tan^{-1}\dfrac{12}{4.9498} = 67.6°$

(d) Let Y be a point on AB such that $VY \perp AB$

$LY = 6\tan 19.4712° = 2.12\text{ cm}$

Required angle $= \tan^{-1}\dfrac{12}{2.12} = 80.0°$

Part 6 – Cut solid

1. (a) $EB = \dfrac{\sqrt{6^2+6^2}}{2} = \dfrac{\sqrt{72}}{2} = 3\sqrt{2} \approx 4.24\text{ cm}$

$EC = \sqrt{(3\sqrt{2})^2 + 6^2} = 3\sqrt{6} \approx 7.35\text{ cm}$

(b) 90°

2. B

3. C

4. (a) Let X be a point on CD such that $BX \perp CD$, and Y be a point on MN such that $BY \perp MN$.

In ΔBCX,

$$BX = \sqrt{5^2 - 1^2} = \sqrt{24} \text{ cm}$$

Reminder:
Prism -> *BCFG* is a rectangle

In $CDMN$,

$$XY = CN = \frac{20}{5} \times 2 = 8 \text{ cm}$$

In ΔBCN,

$$BN = \sqrt{5^2 + 8^2} = \sqrt{89} \text{ cm}$$

In ΔBYN,

$$BY = \sqrt{89 - 1^2} = \sqrt{88} \text{ cm}$$

In ΔBXY,

$$\text{required angle} = \cos^{-1} \frac{8^2 + (\sqrt{88})^2 - (\sqrt{24})^2}{2(8)(\sqrt{88})} = 31.5°$$

(b) Let Z be a point on GH such that $FZ \perp GH$.

In ΔFGZ,

$$FZ = \sqrt{5^2 - 1^2} = \sqrt{24} \text{ cm}$$

$$\text{Required angle} = \tan^{-1} \frac{12}{\sqrt{24}} = 67.8°$$

(c) Extend DA and CB such that they meet at V, note that the required angle is $\angle DVC$.

$$\frac{VA}{VD} = \frac{AB}{DC}$$

$$\frac{VA}{VA + 5} = \frac{3}{5}$$

$$VA = 7.5$$

$$\sin \frac{\angle DVC}{2} = \frac{1.5}{7.5}$$

$$\angle DVC = 23.1°$$

Thus, the required angle $= 23.1°$.

Part 7 – Sliding

1. (a) $OC = \dfrac{\sqrt{9^2 - 4.5^2}}{3} \times 2 = \dfrac{\sqrt{243}}{3}$ cm

Shortest distance $= \sqrt{15^2 - \dfrac{243}{9}} = \sqrt{198} \approx 14.1$ cm

(b) Note that $\tan \angle AHO = \dfrac{OA}{OH}$

Since OA (the height of the pyramid) is a constant, $\tan \angle AHO$ is maximum when OH is minimum. Therefore, $\tan \angle AHO$ is maximum when $AH \perp CD, OH \perp CD$.

Therefore, $\tan \angle AHO$ increases until $AH \perp CD$, then decreases.

2. (a) Let X be a point on AB such that $CX \perp AB, DX \perp AB$

By Heron's formula, area of $\Delta ABC = 20.3332\text{ cm}^2$

$\frac{1}{2} CX(7) = 20.3332$

$CX = 5.8095 \approx 5.81\text{cm}$

$CD = (5.8095)\tan 70° = 15.9614\text{ cm}$

(b) Volume $= \frac{1}{3}(15.9614)(20.3332) = 108\text{ cm}^3$

(c) The volume will decrease since the height of the pyramid decreases when $\angle CXD$ decreases from $70°$ to $30°$.