REGULAR QUIZ 01

Form 6

3D Trigonometry

Part A - MC (@3 marks)

1.	A	$AD = A'D = \sqrt{1.2^2 + 0.8^2} = \sqrt{2.08}$ $AA'^2 = AB^2 + A'B^2 - 2(AB)(A'B)\cos 50^\circ$ $AA' \approx 1.014283828$ $\cos\theta = \dfrac{AD^2 + A'D - AA'^2}{2(AD)(A'D)}$ $\theta = 41.2^\circ$
2.	D	Let X be a point on BE such that $CX \perp BE$ Note that X lies on BM The required angle is $\angle CKX$ $\therefore$ None of the above
3.	B	$CN = \dfrac{1}{2}\sqrt{8^2 + 9^2} = \dfrac{\sqrt{145}}{2}$ $AN = \sqrt{AC^2 + CN^2} = \sqrt{180.25}$ $\sin\theta = \dfrac{4}{\sqrt{180.25}}$ $\theta = 17.3^\circ$
4.	D	Let $2a$ be the side length of the tetrahedron. $4 \times \dfrac{1}{2}(2a)(2a)\sin 60^\circ = 562$ $a^2 = \dfrac{140.5}{\sqrt{3}}$ Height of the tetrahedron $= \sqrt{(\sqrt{3}a)^2 - (\dfrac{\sqrt{3}}{3}a)^2} = \sqrt{\dfrac{8}{3}}a$ Volume $= \dfrac{1}{3}(\dfrac{562}{4})(\sqrt{\dfrac{8}{3}}a) = 689\text{ cm}^3$

5.	A	Let D be a point on AC such that $BD \perp AC$ $\tan\theta = \dfrac{VB}{BD} = 3$ $BD = \dfrac{8}{3}$ $AD = \sqrt{15^2 - (\dfrac{8}{3})^2} = \dfrac{\sqrt{1961}}{3}$ Note that $\Delta ABC \sim \Delta ADB$ $\dfrac{AC}{AB} = \dfrac{AB}{AD}$ $AC = \dfrac{675}{\sqrt{1961}}$ Area $= \dfrac{1}{2}(AC)(BD)$ $= 20.3\text{ cm}^2$
6.	B	Let X be a point on VB such that $CX \perp VB$ and Y be a point on AB such that $XY \perp VB$ By Heron's formula, area of $\Delta VBC = 28.618$ $CX = \dfrac{2(28.618)}{10} = 5.7236$ $BX = \sqrt{6^2 - CX^2} = 1.8$ $\cos\angle VBA = \dfrac{10^2 + 8^2 - 10^2}{2(10)(8)}$ $\cos\angle VBA = 0.4$ $\angle VBA = 66.422^\circ$ $\tan\angle VBA = \dfrac{XY}{BX}$ $XY = 4.1243$ $\cos\angle VBA = \dfrac{BX}{BY}$ $BY = 4.5$ $CY^2 = 6^2 + BY^2 = 56.25$ $\cos\theta = \dfrac{CX^2 + XY^2 - CY^2}{2(CX)(XY)}$ $\theta = 97.889^\circ$ $\therefore$ Required angle $= 97.9^\circ$

1. A 2. D 3. B 4. D 5. A
6. B

Part B - Long Questions (39 marks)

1. (10 marks)

(a) $MB = 48 \times \frac{3}{2+3} = 28.8$ 1M

$\cos \angle VBA = \frac{15}{48} = \frac{5}{16}$ 1M

$MA = \sqrt{AB^2 + MB^2 - 2(AB)(MB)\cos \angle VBA}$ 1M

$MA \approx 34.48825887$

$MA = 34.5$ 1A r.t. 34.5

(b) $\frac{MN}{BC} = \frac{2}{2+3}$

$MN = 12$ 1A

Height of trapezium *MNDA*

$= \sqrt{MA^2 - \left(\frac{DA - MN}{2}\right)^2}$ 1M (either)

≈ 33.29324256

$= 33.3$ 1A

Height of trapezium *MNCB*

$= \sqrt{MB^2 - \left(\frac{BC - MN}{2}\right)^2}$ 1M (either)

≈ 27.35763148

$= 27.4$ 1A

(c) Let θ be the required angle.

$\cos \theta = \frac{33.293^2 + 30^2 - 27.358^2}{2(33.293)(30)}$ 1M

$\theta \approx 50.89388506$

$\therefore$ Required angle $= 50.9^\circ$ 1A r.t. 50.9°

2. (6 marks)

(a) Let Q be a point on VB such that $AQ \perp VB$.

It is trivial that $\Delta VAB \cong \Delta VCB$

$\therefore CQ \perp VB$

Put $s = \dfrac{12+12+8}{2} = 16$

By Heron's formula, area of ΔVAB

$= \sqrt{16(16-12)(16-12)(16-8)}$ 1M

$= \sqrt{2048}$

$AQ = \dfrac{2(\sqrt{2048})}{12} = \dfrac{16\sqrt{2}}{3}$ 1A

$\therefore CQ = AQ = \dfrac{16\sqrt{2}}{3}$

$\cos \angle AQC = \dfrac{AQ^2 + CQ^2 - AC^2}{2(AQ)(CQ)}$ 1M

$\therefore \angle AQC \approx 64.05552023^\circ$

Required angle $= 64.1^\circ$ 1A r.t. 64.1°

(b) $\angle ABC = 60^\circ$

$\cos \angle AVC = \dfrac{12^2 + 12^2 - 8^2}{2(12)(12)}$

$\angle AVC \approx 38.94244127^\circ$

Note that $\sin \dfrac{\angle APC}{2} = \dfrac{\frac{1}{2}AC}{AP}$

Since $AP \geq AQ$, we have $\angle APC \leq \angle AQC$ 1M

$\angle APC$ increases as P moves from B (60°) to Q (64.1°) 1A for correctly describe (f.t.)

and decreases as P moves from Q(64.1°) to V(38.9°).

3. (12 marks)

(a) $BH = DH = 18\cos 70^\circ$ 1M

BD

$= \sqrt{BH^2 + DH^2}$

$= \sqrt{2(18\cos 70^\circ)^2}$ 1M

$\approx 8.706411455\text{ cm}$

$\approx 8.71\text{ cm}$ 1A

(b) Area of ΔACD

$= \frac{1}{2}(18)^2 \sin 140^\circ$ 1M

$\approx 104.1315928\text{ cm}^2$

Volume of the tetrahedron $ABCD$

$= \frac{1}{3} \times \text{area of } \Delta ACD \times BH$

$\approx \frac{1}{3} \times 104.13 \times 18\cos 70^\circ$ 1M

$\approx 213.6906137\text{ cm}^3$

$\approx 214\text{ cm}^3$ 1A

(c) Let h cm be the vertical distance from A to the horizontal ground,

and k cm be the perpendicular distance from A to DC.

Put $s = \frac{BD + BC + CD}{2} \approx 22.35320573$

By Heron's formula, area of ΔBCD

$= \sqrt{s(s - BD)(s - BC)(s - CD)}$

$= \sqrt{22.353(22.353 - 8.7064)(22.353 - 18)(22.353 - 18)}$ 1M

$\approx 76.03165111\text{ cm}^2$

$\frac{1}{3} \times 76.032 \times h = 213.69$ 1M

$h \approx 8.431644345$

$k = 18\sin 40^\circ$ 1M

The required angle θ

$\sin\theta = \frac{h}{k}$ 1M

$\theta \approx 46.78082111^\circ$

$\theta \approx 46.8^\circ$ 2A

The required angle $= 46.8^\circ$

4(a). (11 marks)

(a)(i) $CD = \sqrt{10^2 - 8^2} = 6$ 1M

$BC = \sqrt{8^2 - 6^2} = 2\sqrt{7}$

By Heron's formula, 1M

$s = \frac{8 + 6 + 2\sqrt{7}}{2}$

Area of ΔCAB

$= \sqrt{s(s-8)(s-6)(s-2\sqrt{7})}$

≈ 15.87450787

$= 15.9 \text{ cm}^2$

1A (or $6\sqrt{7}\text{ cm}^2$)

(ii) Volume of pyramid

$= \frac{1}{3}(15.87450787)(6)$ 1M

≈ 31.74901573

$= 31.7 \text{ cm}^3$ 1A r.t. 31.7

(iii) Area of $\Delta ABD = \frac{8 \times 6}{2} = 24$ 1M

Shortest distance

$= \frac{(31.74901573)(3)}{24}$ 1M

≈ 3.968626967

$= 3.97 \text{ cm}$ 1A r.t. 3.97

(iv) $BC^2 + AB^2 = (2\sqrt{7})^2 + 6^2 = 64$ 1M

$AC^2 = 8^2 = 64$

$\therefore BC \perp AB$ and $DB \perp AB$ 1M

$\therefore$ yes, agree. 1A f.t.

4(b). bonus (3 marks)

$DC = \sqrt{10^2 - 6^2} = 8$

$BC = \sqrt{8^2 - 8^2} = 0$ 1M

$\therefore$ It is not possible that $AC = 6$ cm 2A f.t.