SUMMER QUIZ 01

Form 6
AS & GS

Part A - MC (@2 marks)

1.	A	2.	C	3.	B	4.	D	5.	A
6.	A	7.	C	8.	C	9.	B	10.	D
11.	B	12.	D	13.	B				

1.	A	$r=\frac{T(3)}{T(2)}=\frac{360}{480}=\frac{3}{4}$ $T(n)=480\left(\frac{4}{3}\right)\left(\frac{3}{4}\right)^{n-1}$ $=480\left(\frac{4}{3}\right)^2\left(\frac{3}{4}\right)^n$ $=(0.75)^n\left(\frac{2560}{3}\right)$
2.	C	$T(m)\times T(2m+1)=\frac{1}{5}$ $\left(\frac{m-1}{2m}\right)\left(\frac{2m+1-1}{2(2m+1)}\right)=\frac{1}{5}$ $5m-5=4m+2$ $m=7$
3.	B	$T(6)=11$ and $T(8)=-2$ $2d=T(8)-T(6)=-2-11=-13$ $d=-\frac{13}{2}$ $T(2)=T(6)-4d$ $=11-4(\frac{-13}{2})$ $=37$
4.	D	$d=527-550=-23$ By considering $-232,-209,-186,\ldots,527,550$ $-255+23n<0$ $n<11.087$ $\therefore$ 11 terms.

5.	A	$T(18)=S(18)-S(17)$ $=\left[2(18)^2+18\right]-\left[2(17)^2+17\right]$ $=71$
6.	A	$r^2=\frac{T(8)}{T(6)}=\frac{\sqrt{11}}{\sqrt{3}}$ $T(7)\times T(9)$ $=T(6)\times r\times T(8)\times r$ $=T(6)\times T(8)\times r^2$ $=\sqrt{3}\times\sqrt{11}\times\frac{\sqrt{11}}{\sqrt{3}}$ $=11$
7.	C	$a_4=2a_3-a_2$ $53=2a_3-17$ $a_3=35$ $a_5=2a_4-a_3$ $a_5=2(53)-35$ $=71$
8.	C	For I $(4-2b)-(5-2a)$ $(3-2c)-(4-2b)$ $=-1-2(b-a)$ $=-1-2(c-b)$ $=-1-2d$ $=-1-2d$ $\therefore$ I is true. For II b^2-a^2 c^2-b^2 $=(b-a)(b+a)$ $=(c-b)(c+b)$ $=d(2a+d)$ $=d(2a+3d)$ $\therefore$ II is false. For III $(a+c)-b$ $(a+b+c)-(a+c)$ $=a+c-b$ $=b$ $=a+d$ $=a+d$ $\therefore$ III is true.
9.	B	$T(1)=5, T(2)=11, T(3)=19$ $T(10)$ $=5+(6+8+10+\ldots+22)$ $=131$
10.	D	$S(\infty)=\frac{-20}{1-\frac{10}{-20}}=-\frac{40}{3}$

11.	B	$\log 2 - \log 4 + \log 8 - \log 16 + \cdots$ $= (\log 2 - \log 4) + (\log 8 - \log 16) + \cdots$ $= (-\log 2) + (-\log 2) + \cdots$ $= -10\log 2$
12.	D	For I $\frac{b^2}{a^2} = \frac{c^2}{b^2} = \frac{d^2}{c^2} = r^2$ $\therefore$ I is true. For II $\left(-\frac{1}{c}\right) \div \frac{1}{d}$ $\quad \frac{1}{b} \div \left(-\frac{1}{c}\right)$ $\quad \left(-\frac{1}{a}\right) \div \frac{1}{b}$ $= -\frac{d}{c}$ $\quad = -\frac{c}{b}$ $\quad = -\frac{b}{a}$ $= -r$ $\quad = -r$ $\quad = -r$ $\therefore$ II is true. For III $\frac{4+b}{2+a}$ $\quad \frac{8+c}{4+b}$ $\quad \frac{16+d}{8+c}$ $= \frac{4+ar}{2+a}$ $\quad = \frac{8+ar^2}{4+ar}$ $\quad = \frac{16+ar^3}{8+ar^2}$ $\therefore$ III is false. For IV $\frac{b}{9} \div \frac{a}{3}$ $\quad \frac{c}{27} \div \frac{b}{9}$ $\quad \frac{d}{81} \div \frac{c}{27}$ $= \frac{3b}{9a}$ $\quad = \frac{9c}{27b}$ $\quad = \frac{27d}{81c}$ $= \frac{r}{3}$ $\quad = \frac{r}{3}$ $\quad = \frac{r}{3}$ $\therefore$ IV is true.
13.	B	$r^{10-4} = \frac{T(10)}{T(4)} = \frac{\frac{128}{3}}{486} = \frac{64}{729}$ $r = \pm\frac{2}{3}$ For I $\frac{x_5}{x_7} = \frac{1}{r^2} = \frac{9}{4} > 1$

$\therefore$ I is true.

For II

$$S(\infty)=\frac{486\left(\frac{3}{2}\right)^3}{1-\frac{2}{3}}=4920.75<4921$$

$\therefore$ II is true.

For III

$$x_9=\frac{x_{10}}{r}$$

$$x_9=\frac{128}{3}\times\frac{3}{2} \text{ or } x_9=\frac{128}{3}\times-\frac{3}{2}$$

$$x_9=\pm 64$$

$\therefore$ III is false.

Part B - Structural Questions

1. (a) $\dfrac{[2(-63)+(n-1)(14)]n}{2}=525$ 1M

$14n^2-140n-1050=0$

$n=15$ or $n=-5$ (rej.) 1M

$\therefore$ Number of terms = 15 1A

(b) $\dfrac{(-63+m)(15)}{2}=525$ 1M

$m=133$ 1A

(5)

2. $r=-\dfrac{5}{3}$ 1M

$-405\left(-\dfrac{5}{3}\right)^{m-1}>80000000$ 1M

$\left(-\dfrac{5}{3}\right)^{m-1}<-\dfrac{80000000}{405}$

$-\left(\dfrac{5}{3}\right)^{m-1}<-\dfrac{80000000}{405}$ ($\because m-1$ is odd)

$\left(\dfrac{5}{3}\right)^{m-1}>\dfrac{80000000}{405}$ 1M

$(m-1)\log\left(\dfrac{5}{3}\right)>\log\left(\dfrac{80000000}{405}\right)$ 1M

$m-1>23.9$

$m>24.9$

$\therefore m=26$ ($\because m$ is even) 1A

(5)

3. $T(n) = 3n - 53$

$S(n) < 180$

$\dfrac{(-50 + 3n - 53)n}{2} < 180$ 1M

$3n^2 - 103n - 360 < 0$

$-3.20 < n < 37.5$ 1M

$\because n \geq 1$

$\therefore 1 \leq n < 37.5$ 1M can be absorbed

$\therefore$ There are 37. 1A

(4)

4. (a) $\dfrac{ar}{1 - r^2} = 3$ and $\dfrac{ar^2}{1 - r^3} = \dfrac{12}{13}$ 1M

$\dfrac{(1 - r^2)r}{1 - r^3} = \dfrac{4}{13}$ 1M

$\dfrac{(1 + r)r}{1 + r + r^2} = \dfrac{4}{13}$ 1M

$13r + 13r^2 = 4 + 4r + 4r^2$

$9r^2 + 9r - 4 = 0$

$r = \dfrac{1}{3}$ or $r = -\dfrac{4}{3}$ (rej.)

$\therefore$ The common ratio $= \dfrac{1}{3}$ 1A

(b) For $r = \dfrac{1}{3}$,

$\dfrac{a\left(\dfrac{1}{3}\right)}{1 - \left(\dfrac{1}{3}\right)^2} = 3$ 1M

$\therefore a = 8$

$\therefore T(1) + T(2) + T(3) + T(4) + T(5) + \cdots$

$= \dfrac{8}{1 - \dfrac{1}{3}}$

$= 12$ 1A

(c) $A(n)=\log[T(n)]$

$A(n+1)-A(n)$ 1M

$=\log[T(n+1)]-\log[T(n)]$

$=\log\left[8\left(\frac{1}{3}\right)^{n}\right]-\log\left[8\left(\frac{1}{3}\right)^{n-1}\right]$

$=\log\frac{1}{3}=-\log 3$ 1A

$\therefore A(n)$ is an arithmetic sequence. 1A f.t.

(9)

5. (a)(i) The amount he owes $=\$\left(4\times10^{5}\times(1+r\%)-6\times10^{4}\right)$ 1A

(ii) $\left[4\times10^{5}\times(1+r\%)-6\times10^{4}\right]\times(1+r\%)-6\times10^{4}=3.58\times10^{5}$ 1M

$4\times10^{5}(1+r\%)^{2}-6\times10^{4}(1+r\%)-4.18\times10^{5}=0$ 1M

$(1+r\%)=1.1$ or $(1+r\%)=-0.95$ (rej.)

$r=10$ 1A

(4)

(b)(i) The amount he owes

$=4\times10^{5}(1+r\%)^{n-1}-6\times10^{4}\frac{\left[(1+r\%)^{n-1}-1\right]}{\left[(1+r\%)-1\right]}$ 1M

$=\$\left(6\times10^{5}-2\times10^{5}(1.1)^{n-1}\right)$ 1A

(ii) $6\times10^{5}-2\times10^{5}(1.1)^{n-1}<2\times10^{5}$

$2\times10^{5}(1.1)^{n-1}>4\times10^{5}$

$1.1^{n-1}>2$

$(n-1)\log 1.1>\log 2$ 1M

$n>8.272540897\ldots$

$\therefore n=9$

$\therefore$ At the start of 9th year. 1A

(4)

(c) $\begin{cases} a+b(1.21)=126400 \\ a+b(1.21)^2=85744 \end{cases}$

Solving, we have

$a=320000$ and $b=-160000$ 1M

$320000-160000(1.21)^{m-1}<6\times10^5-2\times10^5(1.1)^{m+7}$

$\frac{4}{1.1^2}\left((1.1)^m\right)^2-5\times1.1^7(1.1)^m+7>0$ 1M

$1.1^m<1.240602098$ or $1.1^m>1.706832516$

$m\log1.1<\log1.240602098$ or $m\log1.1>\log1.706832516$

$m<2.262054538$ or $m>5.609467152$ 1M

The amount of he owes for the 2nd car is NOT less than for the 1st car in the 3rd year.

So, incorrect. 1A f.t.

(4)