

AS & GS

Form 6

Vol 1

Part 1 – Basic Concept

2. A

3. D

4. D

5. (a) $T(3) + T(13)$
 $= 17 - 5(3) + 17 - 5(13)$
 $= -46$

(b) $T(n) = -98$
 $17 - 5n = -98$
 $5n = 115$
 $n = 23$

(c) $T(n) = -303$
 $17 - 5n = -303$
 $5n = 320$
 $n = 64$
 \therefore yes, 64th term

6. (a) $T(n) > 0$
 $6n - 333 > 0$
 $6n > 333$
 $n > 55.5$
 \therefore The least value of $n = 56$.

(b) $T(n) = 38$
 $6n - 333 = 38$
 $6n = 371$
 $n = \frac{371}{6}$

Since n is an integer,

\therefore 38 is not a term in the sequence.

7. For $n = 3$,

$$\begin{aligned} a_5 &= a_4 + a_3 \\ &= 17 + 10 \\ &= 27 \end{aligned}$$

For $n = 4$,

$$\begin{aligned} a_6 &= a_5 + a_4 \\ &= 27 + 17 \\ &= 44 \end{aligned}$$

For $n = 5$,

$$\begin{aligned} a_7 &= a_6 + a_5 \\ &= 44 + 27 \\ &= 71 \end{aligned}$$

For $n = 6$,

$$\begin{aligned} a_8 &= a_7 + a_6 \\ &= 71 + 44 \\ &= 115 \end{aligned}$$

For $n = 7$,

$$\begin{aligned} a_9 &= a_8 + a_7 \\ &= 115 + 71 \\ &= 186 \end{aligned}$$

8. For $n = 2$,

$$\begin{aligned} a_4 &= a_3 + a_2 \\ 44 &= a_3 + 8 \\ a_3 &= 36 \end{aligned}$$

For $n = 3$,

$$\begin{aligned} a_5 &= a_4 + a_3 \\ &= 44 + 36 \\ &= 80 \end{aligned}$$

For $n = 4$,

$$\begin{aligned} a_6 &= a_5 + a_4 \\ &= 80 + 44 \\ &= 124 \end{aligned}$$

For $n = 5$,

$$\begin{aligned} a_7 &= a_6 + a_5 \\ &= 124 + 80 \\ &= 204 \end{aligned}$$

For $n = 6$,

$$\begin{aligned} a_8 &= a_7 + a_6 \\ &= 204 + 124 \\ &= 328 \end{aligned}$$

Part 2 – Arithmetic Sequence

4. (a) $T(3) = 29$

$$a + 2d = 29 \dots (1)$$

$$T(6) = 41$$

$$a + 5d = 41 \dots (2)$$

$$\therefore \text{ we have } d = 4, a = 21$$

$$\text{General term} = 4n + 17$$

(b) $T(n) + T(n+1) = 214$

$$4n + 17 + [4(n+1) + 17] = 214$$

$$8n = 176$$

$$n = 22$$

\therefore 22nd term and 23rd term

5. (a) $T(2) = 113$

$$a + d = 113 \dots (1)$$

$$T(10) = 73$$

$$a + 9d = 73 \dots (2)$$

$$\therefore \text{ we have } d = -5, a = 118$$

$$\text{General term} = -5n + 123$$

(b) $T(n) = -32$

$$-5n + 123 = -32$$

$$5n = 155$$

$$n = 31$$

\therefore 31st term

(c) $T(k) < 0$

$$-5n + 123 < 0$$

$$n > 24.6$$

\therefore 25th term

6. (a) $T(6) + T(11) = 86$

$$a + 5d + a + 10d = 86$$

$$2a + 15d = 86 \dots (1)$$

$$T(23) = -15$$

$$a + 22d = -15 \dots (2)$$

$$\therefore \text{ we have } d = -4, a = 73$$

$$\text{General term} = -4n + 77$$

$$\begin{aligned}
 \text{(b)} \quad T(n) &= 1 \\
 -4n + 77 &= 1 \\
 4n &= 76 \\
 n &= 19 \\
 \therefore & \text{ yes, } 19^{\text{th}} \text{ term}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad T(n) &= (n+1)(n+2) - (n+3)(n+4) \\
 T(1) &= (2)(3) - (4)(5) = -14 \\
 T(2) &= (3)(4) - (5)(6) = -18 \\
 T(3) &= (4)(5) - (6)(7) = -22 \\
 T(2) - T(1) &= -4 \\
 T(3) - T(2) &= -4 \\
 T(2) - T(1) &= T(3) - T(2) \\
 \therefore & \text{ AS}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \text{(a)} \quad a &= -2, \quad d = 0 - (-2) = 2, \quad T(n) = 244 \\
 -4 + 2n &= 244 \\
 n &= 124 \\
 \therefore & \text{ 124 terms}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad a &= 13, \quad d = 4 - 13 = -9, \quad T(n) = -149 \\
 22 - 9n &= -149 \\
 n &= 19 \\
 \therefore & \text{ 19 terms}
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad (4a + b) + (2a - 3b) &= 2(3b) \\
 6a - 2b &= 6b \\
 6a &= 8b \\
 3a &= 4b \\
 \text{first term} &= 2a - 3b, \quad d = -2a + 6b \\
 (2a - 3b) + (n-1)(-2a + 6b) &= 3a + 29b \\
 (n-1)(-2a + 6b) &= a + 32b \\
 (n-1)\left[-2\left(\frac{4b}{3}\right) + 6b\right] &= \frac{4b}{3} + 32b \\
 (n-1)\left(\frac{10b}{3}\right) &= \frac{100b}{3} \\
 n &= 11
 \end{aligned}$$

\therefore 11 terms

$$9. \quad a = 3001, \quad d = 2668 - 3001 = -333$$

$$T(n) > 0$$

$$3334 - 333n > 0$$

$$n < 10\frac{4}{333}$$

\therefore 10 terms

$$10. \quad a = 277, \quad d = 254 - 277 = -23$$

$$T(n) > 0$$

$$300 - 23n > 0$$

$$n < 13\frac{1}{23}$$

\therefore The smallest term

$$= T(13)$$

$$= 280 - 23(12)$$

$$= 1$$

Part 3 - Properties of A.S.

$$3. \quad (a) \quad T(1) = 8 \\ a = 8$$

$$T(6) = 20$$

$$8 + (6-1)d = 20$$

$$d = \frac{12}{5}$$

$$\therefore \text{common difference} = \frac{12}{5}$$

$$(b) \quad T(3) = b$$

$$b = 8 + (3-1)\left(\frac{12}{5}\right)$$

$$= \frac{64}{5}$$

$$b^2 = \frac{4096}{25}$$

$$4. \quad (a) \quad -2x + 3x + 2 = 2(-x + 4)$$

$$x + 2 = -2x + 8$$

$$3x = 6$$

$$x = 2$$

(b) $T(1) = 3x + 2 = 8$

$T(2) = -x + 4 = 2$

$T(3) = -2x = -4$

\therefore general term $= 14 - 6n$

Aspire Education