AS & GS

Form 6
Vol 1

Part 7 – Geometric Sequence

1. D

2.
$$\frac{T(2)}{T(1)}=\frac{-14}{7}=-2$$
$$\frac{T(3)}{T(2)}=\frac{28}{-14}=-2$$
$$\frac{T(4)}{T(3)}=\frac{-42}{28}=-\frac{3}{2}$$
$\therefore$ No

3. (a)
$$T(n)=\frac{125}{8}\left(-\frac{2}{5}\right)^{n-1}$$
$$=(-1)^{n-1}\left(\frac{2}{5}\right)^{-3}\left(\frac{2}{5}\right)^{n-1}$$
$$=(-1)^{n-1}\left(\frac{2}{5}\right)^{n-4}$$

(b) $T(5)=(-1)^{5-1}\left(\frac{2}{5}\right)^{5-4}=\frac{2}{5}$

4.

$$T(3)=\frac{1}{27}$$

$$ar^2=\frac{1}{27}\text{......(1)}$$

$$T(6)=-\frac{1}{729}$$

$$ar^5=\frac{1}{729}\text{......(2)}$$

$$\therefore\quad r^3=-\frac{1}{27}$$

$$r=-\frac{1}{3}$$

$$a=\frac{1}{3}$$

general term

$$=\frac{1}{3}\left(-\frac{1}{3}\right)^{n-1}$$

$$=(-1)^{n-1}\left(\frac{1}{3}\right)^{n}$$

5.

$$T(4)=27$$

$$ar^3=27\text{......(1)}$$

$$T(6)=3$$

$$ar^5=3\text{......(2)}$$

$$r^2=\frac{1}{9}$$

$$\therefore\quad r=\pm\frac{1}{3}$$

$$a=\pm\frac{1}{729}$$

case 1: $a=729$, $r=\frac{1}{3}$

general term

$$=729\left(\frac{1}{3}\right)^{n-1}$$

$$=\left(\frac{1}{3}\right)^{n-7}\ (\text{or } 3^{7-n})$$

case 2: $a=-729$, $r=-\frac{1}{3}$

general term

$=-729\left(-\frac{1}{3}\right)^{n-1}$

$=(-1)^n\left(\frac{1}{3}\right)^{n-7}$ (or $(-1)^n(3^{7-n})$)

6. $T(1)\times T(4)=108$

$a^2r^3=108......(1)$

$T(3)=18$

$ar^2=18$

$a^2r^4=324......(2)$

$\therefore$ $r=3$

$a=2$

$T(n)=2(3)^{n-1}$

7. (a) $\frac{T(n+1)}{T(n)}=\frac{\frac{a^{n+2}b}{2}}{\frac{a^{n+1}b}{2}}$

$=a$

$\therefore$ $\frac{T(n+1)}{T(n)}$ is a constant.

(b) $\frac{T(2)}{T(1)}=\frac{9}{3}$

$a=3$

$T(1)=3$

$\frac{3^{1+1}b}{2}=3$

$b=\frac{2}{3}$

$\therefore$ $a=3$, $b=\frac{2}{3}$

8.

$$T(4)+T(5)=48$$
$$ar^3+ar^4=48$$
$$ar^3(1+r)=48 \text{......(1)}$$

$$T(2)+T(3)=3$$
$$ar+ar^2=3$$
$$ar(1+r)=3 \text{......(2)}$$

$$\therefore \quad r^2=16$$
$$r=\pm 4$$

9.

$$T(2)+T(5)=72$$
$$ar+ar^4=72$$
$$ar(1+r^3)=72 \text{......(1)}$$

$$T(5)+T(8)=9$$
$$ar^4+ar^7=9$$
$$ar^4(1+r^3)=9 \text{......(2)}$$

$$\therefore \quad r^3=\frac{1}{8}$$
$$r=\frac{1}{2}$$

$$a\left(\frac{1}{2}\right)^4\left[1+\left(\frac{1}{2}\right)^3\right]=9$$
$$a=128$$

$\therefore$ 1^{st} term $=128$

10. $a=2$, $r=\frac{6}{2}=3$

$$T(n)=486$$
$$2(3)^{n-1}=486$$
$$n=6$$

11. (a) $a=-\frac{5}{4}$, $r=\frac{5}{2}\div\left(-\frac{5}{4}\right)=-2$

General term

$=-\frac{5}{4}(-2)^{n-1}$

$=(-1)^n(5)(2)^{n-3}$

(b) $T(k) \ge 2550$

$(-1)^k(5)(2)^{k-3} \ge 2550$

$2^{k-3} \ge 510$

$(k-3)\log 2 \ge \log 510$

$k \ge 11.994$

$\therefore$ value of $k = 12$

12. (a) $a = \dfrac{8}{81}$, $r = \dfrac{4}{27} \div \dfrac{8}{81} = \dfrac{3}{2}$

General term

$$T(n) = \frac{8}{81}\left(\frac{3}{2}\right)^{n-1}$$

$$= \frac{1}{3}\left(\frac{3}{2}\right)^{n-4}$$

(b) $T(k) < \dfrac{81}{32}$

$$\frac{1}{3}\left(\frac{3}{2}\right)^{k-4} < \frac{81}{32}$$

$$\left(\frac{3}{2}\right)^{k-4} < \frac{243}{32}$$

$$(k-4)\log\frac{3}{2} < \log\frac{243}{32}$$

$$k < 9$$

$\therefore$ greatest value of $k = 8$

Part 8 – Properties of G.S.

1. B
2. D
3. A
4. $y^2 = 32$

   $y = \pm 4\sqrt{2}$

5.

$$T(1) = \frac{1}{54}$$
$$a = \frac{1}{54}$$

$$T(5) = \frac{3}{2}$$
$$ar^4 = \frac{3}{2}$$
$$\frac{1}{54}r^4 = \frac{3}{2}$$
$$r = \pm 3$$

$$\therefore \begin{cases} x = \frac{1}{18} \\ y = \frac{1}{6} \\ z = \frac{1}{2} \end{cases}, \begin{cases} x = -\frac{1}{18} \\ y = \frac{1}{6} \\ z = -\frac{1}{2} \end{cases}$$